Estimate of Convective Velocity in a Supersonic Turbulent Mixing Layer

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I. Introduction

THE estimation of the convective velocity of large-scale structures in a supersonic turbulent mixing layer is a very critical point for the prediction of mixing in supersonic combustors. As a result, Dimotakis[1] showed that the entrainment ratio in a shear layer depends strongly on the convection velocity of the large-scale structures of Brown and Roshko.[2]

A classical way to estimate convection velocities was popularized by Bogdanoff[3] and Papamoschou.[4] Their method assumes that there exists a saddle point between the large structures in the mixing layer and that the two flows coming from each part of the layer reach the stagnation point isentropically. This method works well in subsonic flows (see Dimotakis[5]) but is not adapted to supersonic ones as stated by Papamoschou.[6] A new sketch of the flow has been proposed by Papamoschou[6] and Dimotakis[7] which assumes that eddy shocks are formed on one side of the structure. These eddy shocks produce a disymmetric entropy step because the entropy increases through the shock only on one side of the layer (see Fig. 1).

In Sec. II of this Note, a new method for estimating the convection velocity is proposed. This method uses a shock strength parameter model and is based on the flow configuration and the same basic assumptions as in Dimotakis's flow.[7] In Sec. III the results obtained with this new method will be compared both with direct convection velocity measurements and with estimates given by a turbulent diffusion model. In Sec. IV all these results will be compared and discussed. From this discussion it will appear that two different categories of supersonic mixing layers can be found. The first one, corresponding to nonconfined flows, shows convective velocities close to the isentropic estimates. The second one, corresponding to confined flows, follows quite closely shock strength parameter model estimates.

II. Shock Strength Parameter Model

Using the possible sketch of the flow given by Papamoschou[6] and Dimotakis[7] (Fig. 1), we can assume that the total pressure loss in the shock can be written as

$$R = \frac{\left[1+\left(\frac{\gamma_{min}-1}{2}\right)Mc_{min}^2\right]^{\left(\frac{\gamma_{min}}{\gamma_{min}-1}\right)}}{\left[1+\left(\frac{\gamma_{max}-1}{2}\right)Mc_{max}^2\right]^{\left(\frac{\gamma_{max}}{\gamma_{max}-1}\right)}} \tag{1}$$

where γ_{min} and γ_{max} are the specific heat ratios of the two gases involved in this flow, and Mc_{min} and Mc_{max} are the maximum and minimum convective Mach numbers for this flow configuration. If the shock is on the low-speed side, we can write (where Uc_{is} is the isentropic estimation of Uc)

$$Uc > Uc_{is}$$

$$Mc_{max} = Mc_2 = (Uc - U2)/a2$$

$$Mc_{min} = Mc_1 = (U1 - Uc)/a1$$

If the shock is on the high-speed side, $Mc_{max} = Mc_1$ and $Mc_{min} = Mc_2$.

The ratio R is assumed to be a decreasing function of the compressible dissipation terms. That is, when R decreases, the compressible dissipation will increase. Thus, the study of the evolution of R will help us to show how the compressible dissipation evolves when compressibility increases. Furthermore, knowing R, it is possible to calculate the shock Mach number Ms and the shock strength parameter $X = Ms/Mc_{max}$ as outlined next.

For a given mixing layer ($U1$, $U2$, $M1$, $M2$, $a1$, $a2$, $\gamma 1$, $\gamma 2$), R, Mc_{max}, Mc_{min}, Ms, and X can be obtained for each supposed Uc ranging from $U2$ to $U1$ as follows. If $Uc > Uc_{is}$, then $Mc_{max} = Mc_2 = (Uc - U2)/a2$ and $Mc_{min} = Mc_1 = (U1 - Uc)/a1$. Using Mc_{max} and Mc_{min} in Eq. (1) will give us the value of R corresponding to Uc. From this value we can obtain the shock Mach number Ms from classical shock relations if we assume, as in Dimotakis,[7] that this convected shock wave is a normal shock. The same reasoning will be used when $Uc < Uc_{is}$, except that in this case we will have $Mc_{max} = Mc_1$ and $Mc_{min} = Mc_2$. Thus, it is possible to obtain the evolution of R, X, Ms, and Mc_{max} as a function of Uc in the range from $U2$ to $U1$. These calculations show that the evolution of X as a function of Uc has three extrema: one minimum for $Uc = Uc_{is}$ and two maxima for $Uc1 > Uc_{is}$ and $Uc2 < Uc_{is}$. These latter two solutions ($Uc1$ and $Uc2$) are shock solutions, and they both are possible configurations for the flow. To minimize entropy, we have to choose the solution with the weaker shock corresponding to the higher R. The use of this method in all of the studied mixing layers gives qualitatively the same kind of results with three

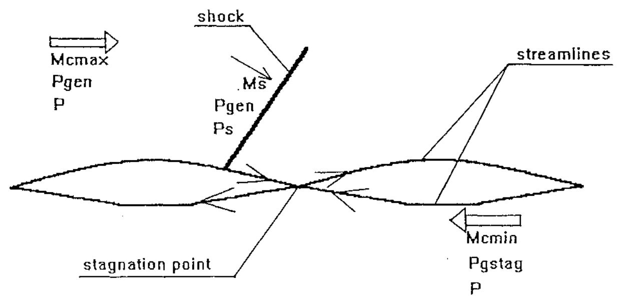

Fig. 1 Possible sketch of the flow (from Ref. 6) in a convective frame of reference linked to large structures: P = mean static pressure in the flow, Ps = static pressure just before the shock, $Pgen$, $Pgstag$ = total pressure for each side of the flow, Mc_{max}, Mc_{min}, Mc = convective Mach numbers.

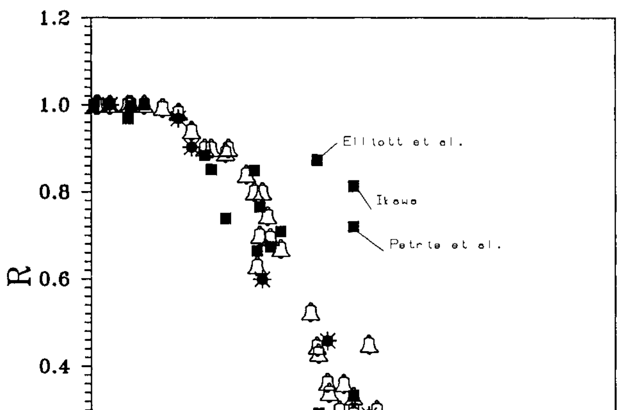

Fig. 2 Evolution of total pressure loss in the shock: △ = estimation with shock strength parameter, ✱ = Papamoschou,[6] ■ = estimation with turbulent diffusion model, and ⍓ = Hall.[19]

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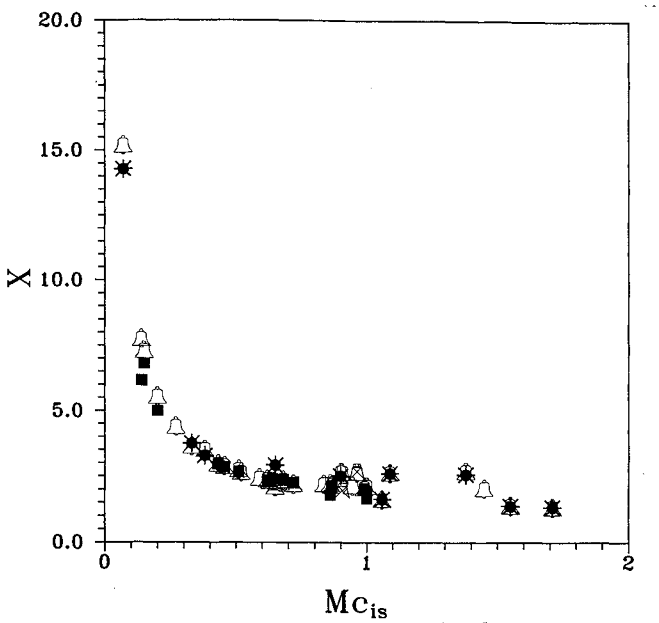

Fig. 3 Evolution of shock strength parameter: △ = estimation with shock strength parameter, ✱ = Papamoschou,[6] ■ = estimation with turbulent diffusion model, and ⩓ = Hall.[19]

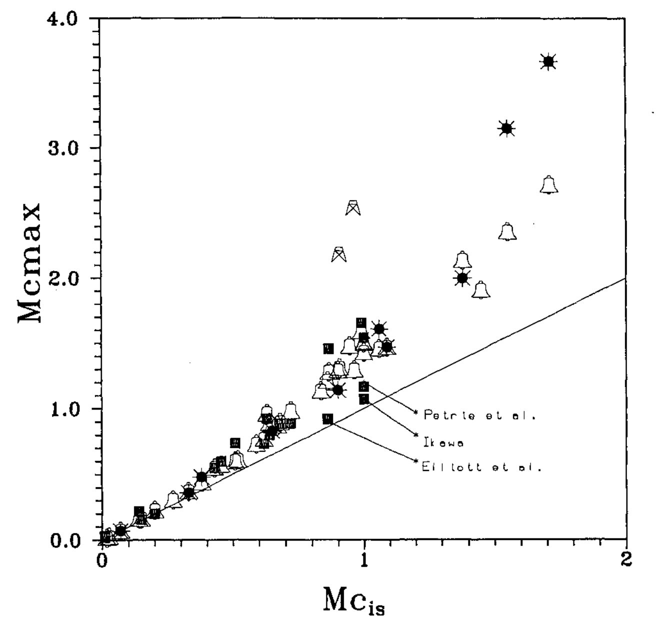

Fig. 4 Evolution of Mc_{max} vs Mc_{is}: △ = estimation with shock strength parameter, ✱ = Papamoschou,[6] ■ = estimation with turbulent diffusion model, and ⩓ = Hall.[19]

extrema on the value of X. The shock is then assumed to be located where the expansion between P and Ps reaches a maximum value corresponding to a maximum on $X = Ms/Mc_{max}$. In fact, X appears to quantify the magnitude of the expansion rate before the shock and then, finally, gives the shock location.

III. Results

This principle was applied to mixing layer data available in the literature (see Refs. 6 and 8–19) that cover a significant range of convective Mach numbers. The results obtained for these flows in terms of R as a function of Mc_{is} where $Mc_{is} = (U_1 - U_2)/(a1 + a2)$ is plotted in Fig. 2. Here we can see both Papamoschou's[6] and Hall's[19] values of R obtained from their direct optical measurements of convective velocity. The present shock strength parameter estimation and a determination of R performed in Barre et al.[20] based on an eddy diffusivity model are also shown in this figure. The calculation with this turbulent diffusion model is not possible for all of the flow cases because it requires knowing the shear stress as an input.

The overall agreement between all of these estimations and direct measurements seems good except for the three cases labeled in Fig. 2 corresponding to Refs. 14, 15, and 17. The possible reasons for these discrepancies will be discussed in Sec. IV.

Figure 3 shows the evolution of the shock strength parameter X as a function of Mc_{is}. Here, the collapse seems excellent for all methods. We can see that the shock strength parameter evolution is very continuous in the range of transonic convective Mach numbers and seems to stabilize near a value of 1.8–2 for the supersonic cases. It also seems that an asymptotic value of the expansion rate before the shock is reached when the considered flow becomes sufficiently compressible.

Figure 4 shows the maximum convective Mach number obtained from each method. The regular evolution of Mc_{max} with Mc_{is} suggested by both direct measurements and indirect estimates shows that there is a gradual increase of the influence of compressibility effects that can be well quantified by the proposed shock model. The discrepancy between Hall's results and the others will be also discussed in Sec. IV.

IV. Discussion

From Fig. 2 we can classify our results into two different families. The first one corresponds to most of the mixing layers studied here (all references except Refs. 14, 15, 17, and 19). In these flows, the effects of compressibility seem to be adequately described by the present shock strength model because the convection velocities given by this model are in good agreement with direct measurements. All of the mixing layers corresponding to this family were realized in relatively small wind tunnels. The ratio δ/D, where δ is some measure of the average mixing layer thickness in the test section and D is the test section height, is of the order of 0.1 for these cases. With these conditions, numerical simulations performed by Si-Ameur et al.[21] showed that it is easier to produce eddy shocks in confined flows than in nonconfined ones. Their direct simulations showed that the two-dimensional character of turbulent perturbations seems to be enhanced by confinement, and hence the shock wave formation can be more easily observed in confined flows rather than in nonconfined cases.

From turbulent diffusion model estimates, the second family (Refs. 14, 15, and 17) seems to have a more "isentropic" behavior. For this reason the shock strength parameter is not able to describe them. For the flow of Petrie et al.,[17] which is a confined one ($\delta/D \gg 0.1$), the turbulent diffusion model estimation of the convective velocity gives a nearly isentropic result; however, no definite conclusion can be made since no direct measurement was performed in this flow. Furthermore, this experiment is done on a reattaching free shear layer that perhaps does not have all of the properties of a completely developed self-similar mixing layer.

For the flows of Ikawa[15] and Elliott and Samimy,[14] the convection velocity determined from the turbulent diffusion model is in good agreement with the direct measurements performed by these authors, as can be seen in Table 1.

Consequently, in these less confined flows compressibility effects do not act in the same way that they do in confined ones, and the shock strength parameter results do not agree with the turbulent diffusion model and direct measurements.

Now, if we consider the direct convective velocity measurements of Hall,[19] we can see that they are out of the range corre-

Table 1 Comparison of different estimates of the convective velocity

	Mc_{is}	Uc_{is},[a] m/s	Uc_k,[b] m/s	Uc_m,[c] m/s
Ikawa[15]	1	344	367	364.5
Elliot and Samimy[14] and Samimy et al.[23]	0.86	427	413	436

[a] Uc_{is} = isentropic estimate.
[b] Uc_k = turbulent diffusion model estimate.
[c] Uc_m = measured values.

sponding to the two families described earlier. They do not agree with either the shock strength model estimation or the turbulent diffusion model as shown by Figs. 2 and 4. A possible explanation for this discrepancy is that Hall had measured the velocity of the source radiating into the outer flow by the mixing layer, which is not the large eddy convection velocity. The measurements made at IMST by Barre[22] confirm that *Uc* is different from the sound sources' velocity *Us*.

In conclusion, a new method, based on a shock strength parameter, was used to calculate the convective velocity of large-scale structures in a specific flow configuration corresponding to an assumption of convected shock waves linked to the eddies. From all of the results presented in this note, it appears that this assumption seems to adequately describe a large number of flows where the confinment ratio δ/D is of the order of 0.1. It has been shown that this model cannot describe nonconfined flows, which seem to behave in a more isentropic way.

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Quasiglobal Reaction Model for Ethylene Combustion

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Introduction

THE current interest in hypersonic flight has created a resurgence of interest in hypersonic airbreathing propulsion systems. In this flight regime, most of the work has concentrated on hydrogen-fueled supersonic combustion ramjet engines. For a Mach number range of 4–10, hydrocarbon fuels provide sufficient thrust and are also being considered. The high density of hydrocarbon fuels, as well as ease of handling, makes them very attractive for volume-constrained systems such as missiles and the hypersonic research vehicle (HRV).

The design of future hypersonic propulsion systems will depend very heavily on computational fluid dynamics (CFD) because of the difficulties associated with testing combustors in ground-based facilities at flight speeds. Therefore, it is essential to develop CFD codes capable of numerically simulating the hostile environment of a combustor. The numerical simulation of conservative equations, including a detailed kinetics system for a multidimensional system, is computationally prohibitive, if not impossible. Even with the availability of supercomputers, routine computations of combustor flowfields are not possible. Most of the computer codes that include detailed chemical kinetics are limited to one dimension. In order to help CFD design the engine, a real need exists for the development of reduced mechanisms. In this study, attention will be focused on ethylene (C_2H_4). Ethylene is chosen because it is used as a surrogate test fuel for hydrocarbon fuels. Also, because ethylene is usually an intermediate product in the combustion of heavy hydrocarbons, the developed model can also be used for assembling reaction mechanisms of heavy hydrocarbons such as propane, butane, n-heptane, etc. Westbrook and Dryer[1] developed a two-step reaction mechanism for ethylene. The assumption of CO, CO_2, and H_2O as the final products overpredicts the heat of reaction; hence, a higher flame temperature results. This model was developed primarily for laminar flame calculations and would not be adequate for combustor application.

The objective of this study is to develop a reduced mechanism for ethylene oxidation. We are interested in a model with a minimum number of species and reactions that still models the chemistry with reasonable accuracy for the expected combustor conditions. The model will be validated by comparing the results to those calculated with a detailed kinetic model that has been validated against the experimental data. The detailed ethylene mecha-

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